Some Notes on the Use of the Watts Limb-Correction Charts

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(Received 11 May 1970)

Grazing and total occultations of stars by the moon have been used to solve for parameters to refine limb-correction data read from Watts' charts of the moon's marginal zone. When using the charts, the following parameter values and standard errors are indicated by this discussion: correction to adopted radius of the datum, $-0''.022\pm0''.025$; ellipticity (spherical datum minus Watts datum), $-0''.15\cos 2(\mathrm{AA}-153°)$; correction to Watts angles (WA) to get axis angles (AA), $-0°.22\pm0°.02$; longitude and latitude components of Watts center minus center of mass, $+0''.25\pm0''.74$ and $-0''.24\pm0''.10$, respectively; radial component (correction to constant of sine parallax), $+0''.075\pm0''.047$.

I. INTRODUCTION

REDUCTIONS of grazing and total occultations of stars by the moon have been greatly aided by the availability of corrections for limb irregularities from the charts of the marginal zone of the moon compiled by C. B. Watts (1963). These corrections have led to reductions of typical total occultation residuals by a factor of 2, to the removal of many important sources of systematic error in the determination of lunar elements, and to the possibility of observing and reducing extreme grazing occultations.

The Watts charts enjoy several important advantages over earlier and contemporary charts. For instance, they are presented in the form of charts of the libration frame for each 0.2 deg of position angle measured from the moon's axis of rotation (projected onto the celestial sphere), hereafter referred to as "axis angle." Thus, for any given axis angle, limb corrections for any given values of the longitude and latitude librations may be read. This means that oblique, as well as normal, gradients across the marginal zone of the moon's limb are represented.

Perhaps the single most important feature of the charts is their referral to a spherical datum. Even though the charts were compiled from a large number of circular arcs from photographs of the moon's limb, every pair of such arcs must have at least one point in common on the illuminated limb. Hence, by careful intercomparison of all of the arcs near the common points, a spherical reference datum, with center fixed in the moon (although not necessarily at the center of mass), may be built up.

II. RADIUS OF THE DATUM

Because of irradiation and scale problems, the absolute radius of this spherical datum could not be determined during construction of the charts. Occultations observed at the moon's dark limb (both disappearances and reappearances), especially photoelectric observations, enable a relatively error-free value for the radius of this datum to be determined. The radius of the bright limb of the moon, as observed visually or photographically, would be somewhat larger, depending upon observer and magnification, because of irradiation. In this investigation the assumed value for the datum radius was 0.2725026 earth radii, with an accompanying assumed constant of sine parallax of $3422''.451$, and earth radius of 6378160 meters. The occultations give a correction of $-0''.022\pm0''.025$ (standard error) to the assumed radius (divide by $3422''.451$ to express in earth radii). See Sec. VI for the accompanying correction to parallax.

Note that the radius of the Watts datum, which is a mean profile radius, bears no simple relationship to the mean radius of the moon, from its center to its physical surface. This is because the limb projected onto the celestial sphere is a profile, and not a representation of the physical surface. Any point on the profile is the highest point that results from projecting foreground and background features onto the limb. Hence, any low surface region, surrounded by higher regions, will never contribute to the profile. The rougher the surface, the greater the excess of profile radius over physical radius.

III. ELLIPTICITY OF THE DATUM

Watts expected that his datum might turn out to be slightly elliptical because of the weak connection near the lunar poles between measures of the east and west limbs. He solved for such an ellipticity experimentally, by constructing artificial moon images. His result was a correction of

$$+0.15\cos 2(\mathrm{AA}-153°),$$

where AA is axis angle, which is incorporated in the published charts. The standard error of the coefficient is $\pm0''.12$, and that of the AA of the major axis is $\pm22°$.

The occultation observations were solved for the ellipticity of the datum. The results are a correction to the Watts datum of

$$-0''.164\cos 2(\mathrm{AA}-147°.0),$$

with standard errors of $\pm0''.018$ and $\pm4°.4$, respectively. Because of the unmistakable similarity of this correction to the one which Watts applied, it is assumed here that the Watts correction for ellipticity should be removed in its entirety, and that the remaining ellipticity is not statistically significant. Hence, any reading from the charts should be amended by the amount

$$-0''.15 \cos 2(\mathrm{AA} - 153°).$$

IV. AXIS ANGLES

Early reduction of grazing occultations indicated that, while all major features represented in the Watts charts were also present in the observations, there was a shift of between 0.1 and 0.3 deg in axis angle between the two representations of limb features. This shift turned out to be always in the same direction, for both north and south limb grazes, in the sense that the axis angles of features in the Watts charts (to be referred to hereafter as "Watts angles," WA) were 0.2 deg larger on the average than the axis angles of the same features derived from observations (AA). Hence, AA = WA − 0°.2.

It was not known for some time whether the amount was constant or time varying, whether it was the same all around the limb or only near the poles, or whether the problem originated in the reduction method or the charts. Recent studies of total occultations have helped to answer these questions.

Limb corrections for total occultations were read for the predicted axis angle of occultation and for neighboring points, so that partial derivatives of the occultation residuals with respect to axis angle could be formed. Analysis of the results indicates a possible time variation of the axis angle correction with a period not well determined, but possibly a little less than a year. This might be due only to the influence of changing libration combinations, however. Similar variations have been found by R. Abileah from studies of grazing occultation data received by the U. S. Naval Observatory. The time variation, if it is real, is not yet well described.

Initially, the total occultations indicated a smaller mean value, 0.1 deg, for the axis angle shift averaged over large arcs of the moon's leading and trailing limbs. It was eventually discovered that linear partial derivatives were inadequate in solving for this correction over an interval as large as 0.2 deg. Moreover, errors introduced during the limb correction look-up process had the tendency to reduce the axis angle correction to zero from whatever value might be assumed in the analysis. Hence, an iterative solution had to be used, with the result that 0.22±0.02 deg was found to be appropriate for the entire limb.

An independent investigation using grazing occultation data (Morrison 1970) resulted in an estimate of 0.25±0.01 deg for the axis angle shift. This investigation produced one other important result—a comparison of the Watts charts with those of Weimer (1952) for selected features showed essentially the same shift. This is a strong indication that the shift is not caused by some oversight in the occultation reduction procedure.

Extensive investigations into both the reduction method and the chart construction were carried out in an effort to discover the cause of this discrepancy. Much of the latter work was carried out by Joan E. Bixby at the U. S. Naval Observatory. The eventual outcome was the discovery of a correction ranging from 0.09 to 0.11 deg to the Watts angles which was applied with the wrong sign. This is indeed a fortunate resolution of the problem. Finding a nearly constant correction eliminates the need to worry about any effects on the elevations or possible distortion of the datum. Moreover, the correction may be easily applied, since it is very close to the chart interval unit in axis angle.

V. OPTICAL LOCATION OF CENTER OF DATUM

In principle, the latitude component of the center of mass to center of Watts datum correction is easily measurable in occultation observations, being reflected in any failure of the mean latitude to be zero. In practice, one must separate it from systematic errors in the star positions, especially a correction to the equator point of the fundamental coordinate system. When this is done, the occultations give a correction of $-0''.24 \pm 0''.10$ at mean distance, in the sense that the center of the Watts datum is below the center of mass.

Finding the corresponding correction for longitude is a much more complex problem. The principal effect on the observations of such a correction is due to the 1-1/2-deg inclination of the moon's axis of rotation to the ecliptic. But this effect is nearly identical to the effect of a correction to the nodes of the moon's orbit, so that the two causes cannot be easily separated.

A very indirect measure of the correction may be obtained by comparing direct observations of the longitudes of the geometric (or Watts) centers of the sun and moon with a correction to the mean elongation of the sun from the moon, which is an argument in the lunar theory. The latter angle depends upon the location of the centers of mass of the two objects. Assuming coincidence of the sun's center of mass and geometric center (because of its symmetry), we can assign any discrepancy between the two results to a displacement of the moon's center of mass from the Watts datum center. A problem arises from the correlation of corrections to mean elongation with corrections to perigee, but the correlation is not hopelessly large. An estimate for the longitude correction using this method is $+0''.25 \pm 0''.74$ at mean distance, in the sense that the Watts datum center leads the center of mass in the moon's orbit. (Note: It is assumed here that this correction is uncomplicated by planetary aberration, which is missing from the elements of the moon's orbit, but compensated for in the theory itself.)

As a consequence of the inclination of the moon's axis to the ecliptic (1-1/2 deg), the displacement of the centers in longitude (δL) will cause a periodic latitude variation $\delta\beta = +0.0268 \cos F(\delta L)$, where F is the moon's argument of latitude. Similarly, the latitude displacement (δB) causes a periodic longitude variation

$\delta\lambda = -0.0268 \cos F(\delta B)$. The latitude variation would be completely taken up into a correction to the nodes ($\delta\Omega$), but the longitude variation would not be so absorbed. To remove the effect of $\delta\beta$ on $\delta\Omega$, the correction is $\delta(\delta\Omega) = +0.30(\delta L)$.

VI. RADIAL LOCATION OF CENTER OF DATUM

The direct solution for a displacement of the center of the Watts datum along the mean Earth–moon radius vector involves periodic terms in longitude with the moon's anomalistic period, and in latitude with the moon's nodal period. These are almost completely inseparable from corrections to the moon's eccentricity and inclination, respectively. Instead we may use the topocentric displacement to solve for the distance, or equivalently the constant of sine parallax, of the center of the Watts datum. The constant of sine parallax for the center of mass, derived from its theoretical relationship to the moon's mean motion and the Earth's equatorial radius, is $3422''.451$. The observed correction from the occultation data is $+0''.075 \pm 0''.047$ at mean distance, in the sense that the Watts center is closer to the Earth than the center of mass.

A consequence of this radial displacement, which is about 8.4 km, is the introduction of periodic variations into longitude and latitude, similar to those introduced by δL and δB. If we denote this radial component by $\delta\pi$, then the resulting variations will be $\delta\lambda = 60.3 \sin l'(\delta\pi)$ and $\delta\beta = -60.3 \sin b'(\delta\pi)$, where l' and b' are the topocentric longitude and latitude librations, respectively. Since the librations have the moon's anomalistic and nodal periods, these variations will be almost completely taken up into corrections to eccentricity (δe) and inclination (δI), respectively. To remove the effects on δe and δI, the corrections are $\delta(\delta e) = +3.3(\delta\pi)$ and $\delta(\delta I) = +5.4(\delta\pi)$.

It should be noted that each of the variations described in Secs. V and VI is magnified in the ratio of the mean distance to the true distance of the moon, introducing still more periodic variations.

VII. CONCLUSION

The corrections and parameters discussed in this paper are found to reduce residuals in the comparison of occultation observations with the lunar ephemeris, using limb corrections derived from Watts' charts. They should in no sense be construed as criticism of the charts. Rather, the analysis on which this paper is based should be considered as a full confirmation of the stated accuracy of the charts, namely a probable error of $\pm 0''.10$ for a typical reading. Furthermore, one should remember that a reduction in the residuals by a factor of 2 is made possible by the availability of the Watts charts, while the refinements discussed here cause a further reduction of less than 10% in the residuals. A discussion of lunar observations 24 years ago (Brouwer and Watts 1946), revealed values for corrections derived from various sources that were inconsistent by 20 to 30 times their probable errors. That discussion provided much of the incentive for undertaking the construction of the Watts charts, a project which consumed 15 years. The indebtedness of fundamental astronomy to Dr. Watts is very great, indeed!

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